

Testing the shoulder condition in Point Transect sampling

*Un test sulla “shoulder condition”
nel campionamento “Point Transect”*

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Riassunto. I più comuni stimatori della densità di una popolazione biologica basati sul campionamento per transetti sono caratterizzati da un'eccessiva sensibilità rispetto alla violazione della condizione di stazionarietà della funzione di avvistamento (“shoulder condition”). Nasce da qui l'esigenza della verifica di tale condizione. L'obiettivo del lavoro consiste nel proporre un test sulla “shoulder condition” nel campionamento “Point Transect”. Si tratta del test uniformemente più potente tra i test invarianti rispetto ai cambiamenti di scala per verificare l'appartenenza della funzione di avvistamento alla famiglia seminormale, contro l'appartenenza alla famiglia esponenziale negativa. Queste famiglie rappresentano due situazioni comuni nelle applicazioni, ma ben distinte dato che la seconda, a differenza della prima, non soddisfa alla “shoulder condition”.

Keywords: Point Transect Sampling, Shoulder Condition, Uniformly Most Powerful Invariant Test, Asymptotic Critical Values, Monte Carlo Critical Values.

1. Introduction

Many studies of wildlife populations require estimates of population abundance. Point transect sampling provides an effective method for estimating population size ν or density $\delta = \nu / A$, where A is the area of the study region. According to this design, k points are randomly chosen within the study area and, at each of the selected points, an observer measures the distance from himself to any animal detected.

Since the number of animals observed from each point is small in many contexts where this sample scheme is adopted (as for instance in ornithology), sampled distances are pooled together to increase the sample size.

Let $z_1, \dots, z_n$ be the sample of size n obtained by pooling together the distances measured at each of the k observation points. Let f be the probability density function (pdf) of the observed distances and let g be the detection function, that is to say $g(y)$ is the conditional probability of detecting an animal given that it is at distance y from the observer. It turns out that the relation

$$f(z) = \frac{zg(z)}{\int_0^{+\infty} yg(y)dy} \quad (1)$$

holds for every distance z .

The estimator of the population density δ is $\hat{\delta} = \frac{n}{2k\pi} \hat{f}'(0)$ where $\hat{f}'(0)$ is an estimator of the derivative of f at 0 which satisfies the fundamental identity

$$f'(0) = \frac{1}{\int_0^{+\infty} yg(y)dy}$$

(Buckland et al., 2001). So the basic problem for estimating δ , or equivalently ν , is to estimate $f'(0)$.

In a previous paper (Borgoni et al., 2005) we investigated the small-sample behaviour of different estimators of δ depending on the shape of the detection function. We considered the two most popular families of detection functions (Zhang, 2001):

the half-normal family

$$g(y) = \exp\left(-\frac{y^2}{2\sigma^2}\right) \quad (\sigma > 0) \quad (2)$$

and the negative exponential family

$$g(y) = \exp\left(-\frac{y}{\sigma}\right) \quad (\sigma > 0). \quad (3)$$

The former family satisfies the shape criterion

$$g'(0) = 0 \quad (4)$$

whilst the latter does not. This property, also known as shoulder condition, ensures that animal detection is nearly certain at small distances from the observer (Buckland et al., 2001; Buckland et al., 2004). However such a condition fails when detectability decreases sharply around the observation points because of bad visibility (e.g. in presence of fog or dense vegetation).

Borgoni et al. (2005) showed some simulation results suggesting that the usual estimators of δ are extremely sensitive to departures from the shape criterion. Testing such a hypothesis is therefore a preliminary step for any attempt to estimate wildlife population density via transect sampling (Zhang, 2003).

The aim of this paper is to propose a procedure for testing the shoulder condition (4). In particular we focus on a test for discriminating the two families (2) and (3). In view of (1) this turns out to be equivalent to testing that the distance pdf belongs to one of the two following families:

$$F_0 = \left\{ f(z) = \frac{z}{\sigma^2} \exp\left(-\frac{z^2}{2\sigma^2}\right) : \sigma > 0 \right\} \quad (5)$$

and

$$F_1 = \left\{ f(z) = \frac{z}{\sigma^2} \exp\left(-\frac{z}{\sigma}\right) : \sigma > 0 \right\}. \quad (6)$$

The proposed test is the uniformly most powerful (UMP) in the class of the scale invariant tests. The asymptotic distributions of the test statistic under (5) and (6) are worked out. The critical values and the powers of the test are tabulated via Monte Carlo simulation for several typical α -levels and small sample sizes n .

2. The UMP scale invariant test

Given n independent observations $z_1, \dots, z_n$ from an unknown pdf f , we consider the problem of testing

$$H_0 : f \in F_0 \quad \text{vs.} \quad H_1 : f \in F_1, \quad (7)$$

where F_0 is the family of Rayleigh distributions with scale parameter σ and F_1 is the family of Gamma distributions with shape parameter 2 and scale parameter σ , specified in (5) and (6) respectively.

This problem is invariant under the group of scale transformations

$$G = \{g(z) = rz : r > 0\}.$$

A maximal invariant under G (Lehmann and Romano, 2005) is

$$(z_1/z_n, z_2/z_n, \dots, z_{n-1}/z_n).$$

It can be shown that the UMP test among functions of this maximal invariant rejects the null hypothesis for large values of the likelihood ratio

$$\lambda = \frac{(2n-1)!}{2^{n-1}(n-1)!} \left[\frac{\sum_{i=1}^n z_i^2}{\left(\sum_{i=1}^n z_i\right)^2} \right]^n.$$

Since λ is a monotonically increasing function of the statistic

$$Q_n = \frac{1}{n} \sum_{i=1}^n z_i^2 \bigg/ \left(\frac{1}{n} \sum_{i=1}^n z_i \right)^2, \quad (8)$$

the critical region of the UMP scale invariant test for the hypotheses (7) can be written as

$$Q_n \geq q_{n,\alpha}, \quad (9)$$

where α denotes the level of significance and $q_{n,\alpha}$ is the corresponding critical value such that

$$P(Q_n \geq q_{n,\alpha} | H_0) = \alpha.$$

Furthermore the asymptotic normal distribution under H_0

$$\sqrt{n/(256/\pi^3 - 80/\pi^2)}(Q_n - 4/\pi) \xrightarrow{d} N(0,1)$$

and under H_1

$$\sqrt{4n/3}(Q_n - 3/2) \xrightarrow{d} N(0,1)$$

follows from the bivariate central limit theorem and the delta method. For large n approximate critical value and power are given by $q_{n,\alpha} \cong 1.27 + 0.39 z_{1-\alpha}/\sqrt{n}$ and $1 - \beta = P(Q_n \geq q_{n,\alpha} | H_1) \cong 1 - \Phi(0.22 z_{1-\alpha} - 0.26\sqrt{n})$, where $z_{1-\alpha}$ is the $(1-\alpha)$ th quantile and Φ is the cumulative distribution function of the standard normal distribution. Hence the proposed test turns out to be consistent (Lehmann, 2001).

As the distribution of the test statistic under (5) or (6) does not depend on the scale parameter, Monte Carlo simulations can be performed in order to obtain the empirical critical values and the powers for small sample sizes (reported in Table 1). The simulation design consists in randomly drawing n distances from the distributions (5) and (6) assuming, without loss of generality, $\sigma = 1$. The statistic (8) is then applied to each of the simulated samples and the procedure is repeated 5000 times. Finally the Monte Carlo approximations of critical value $q_{n,\alpha}$ and power $1-\beta$ are calculated.

3. Conclusions

In transect sampling the problem of testing the shoulder condition of a detection function is invariant under the group of scale transformations. So the scale invariance

provides a natural restriction to impose on the statistical procedure that one wants to use. As the families (2) and (3) are the two most commonly used models of detection functions, the above problem is reduced to (7). Thus the optimal solution is given by the UMP scale invariant test (9). In addition the limiting normal distribution of the test statistic (8) is provided and the consistency of the proposed test is pointed out.

Approximate critical values for some typical sample sizes and levels of significance are carried out by Monte Carlo simulations.

The power under (6) is reasonably good even in the case of a moderate small sample and low α . Although we do not report the results in detail here, we found that the test has a good performance in terms of the power also in the case that the data come from a mixture of (5) and (6). For example for $\alpha = 0.05$ and $n = 40$ we obtained a power of 0.80, 0.75 and 0.54 assuming a contamination proportion of 25%, 50% and 75% respectively. For a slightly larger sample ($n = 50$) the power in the three cases above was 0.87, 0.81 and 0.60 respectively.

Table 1: Empirical critical values $q_{n,\alpha}$ and corresponding power $1-\beta$ (in parentheses) of the UMP scale invariant test.

	$n = 30$	$n = 40$	$n = 50$	$n = 60$
$\alpha = 0.01$	1.46 (0.47)	1.43 (0.61)	1.41 (0.71)	1.40 (0.78)
$\alpha = 0.05$	1.39 (0.68)	1.37 (0.80)	1.37 (0.87)	1.36 (0.91)
$\alpha = 0.10$	1.36 (0.79)	1.35 (0.87)	1.34 (0.92)	1.33 (0.95)

References

- Borgoni R., Cameletti M., Quatto P. (2005) Comparing estimators of animal abundance: a simulation study, Atti del Convegno della Società Italiana di Statistica “Statistica e Ambiente”, Università degli Studi di Messina, 181-184.
- Buckland S.T., Anderson D.R., Burnham K.P., Laake J.L., Borchers D.L., Thomas L. (2001) *Introduction to Distance Sampling*, Oxford University Press, Oxford.
- Buckland S.T., Anderson D.R., Burnham K.P., Laake J.L., Borchers D.L., Thomas L. (2004) *Advanced Distance Sampling*, Oxford University Press, Oxford.
- Lehmann E.L. (2001) *Elements of Large-Sample Theory*, Springer, New York.
- Lehmann E.L., Romano J.P. (2005) *Testing Statistical Hypotheses*, Springer, New York.
- Zhang S. (2001) Generalized likelihood ratio test for the shoulder condition in line transect sampling, *Communications in Statistics – Theory and Methods*, 30, 2343-2354.
- Zhang S. (2003) A Note on Testing the Shoulder Condition in Line Transect Sampling, *Proceedings of the 2003 Hawaii International Conference on Statistics and related topics*.